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Orbits and Revolutions

When a body in space is moving about a primary body, such as the earth, under the influence of gravitational force alone, its path is called its *orbit*. (Figure 1) If a spacecraft is traveling along a closed path with respect to the primary body, its orbit will be a circle or an ellipse. Perfectly circular orbits are not achieved in practice. However the ellipse, as one learns in analytical geometry, approaches a circle when the eccentricity becomes small. The orbits of the manned satellites launched by NASA in the Mer-

cury and Gemini programs were nearly circular — they were ellipses with very small eccentricity. The planets, with the sun as the primary body, also follow nearly circular orbits. When a satellite makes a full trip in orbit around its primary body, it is said to complete a *revolution*, and the time required is termed its *period*, or *period of revolution*.

If an observer is measuring the period, the measurement that he gets will depend upon his point of reference. If he were located far out in space, he

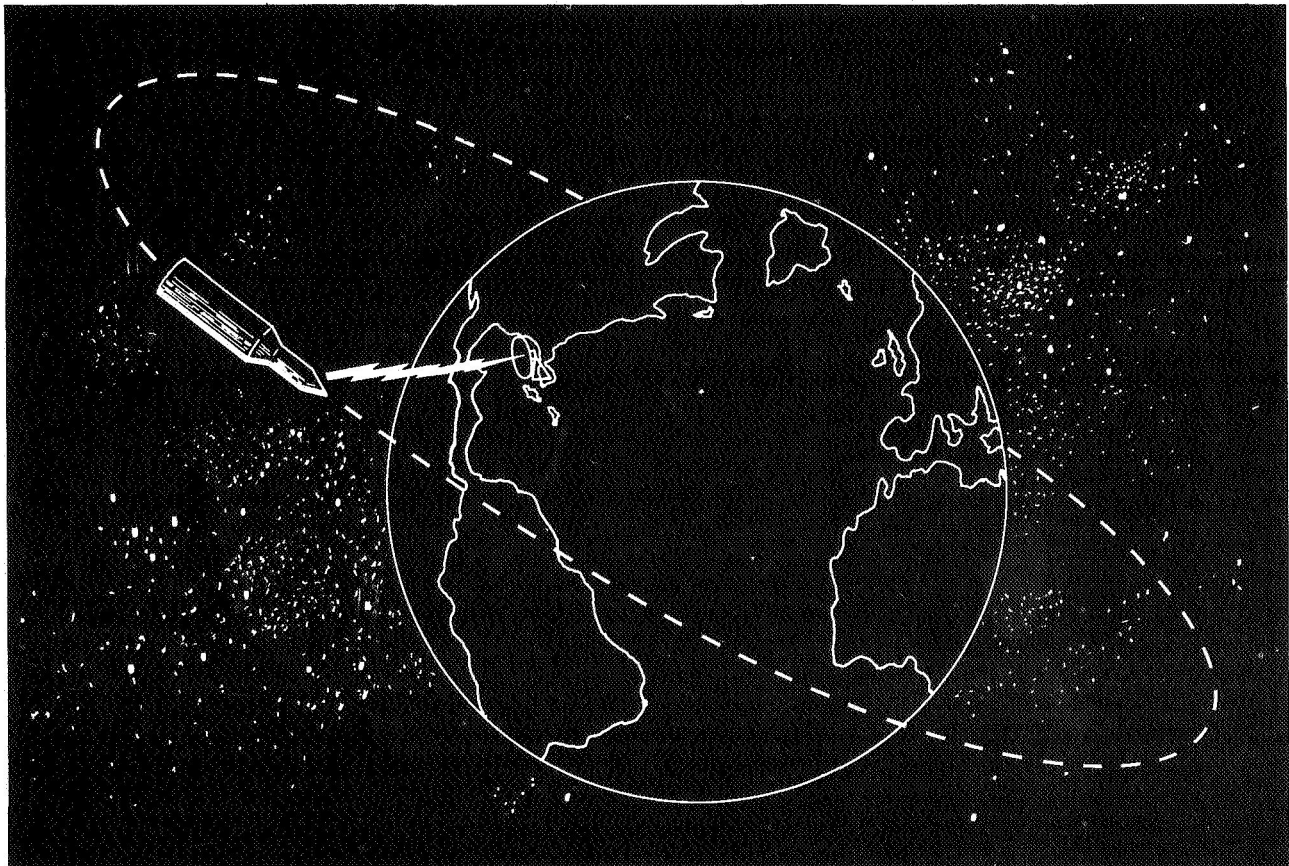


Figure 1

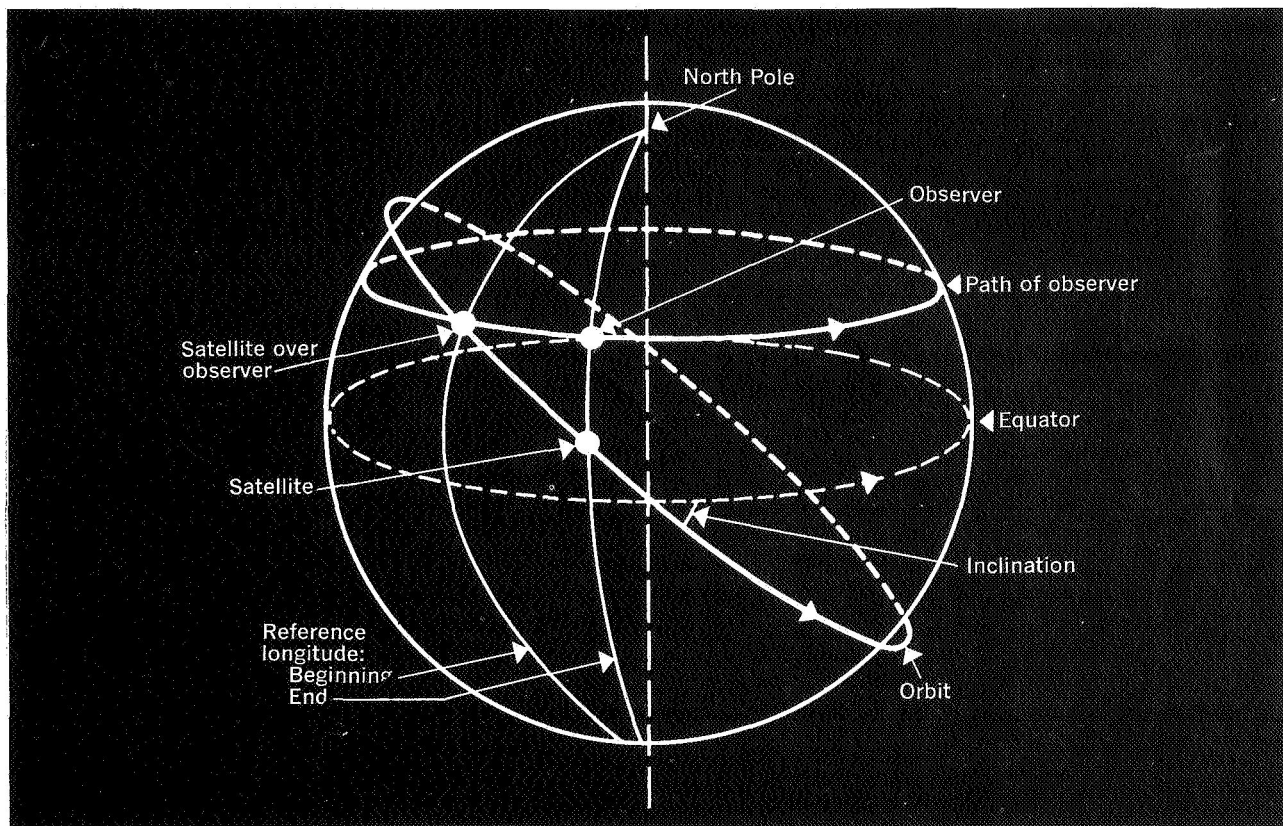


Figure 2. Relative positions of satellite and observer at beginning and end of synodic period

could visualize the orbit of the spacecraft against the background of fixed stars, and determine the period by timing the interval between successive passages over some point in that background. His measurement would then indicate the time needed to make one complete transit of the ellipse and would be called the *sidereal* period of revolution. The word *sidereal* means "of or relating to the stars." The sidereal period is not affected in any way by the rotation of the earth under the satellite.

Let us suppose, however, that the observer is not out in space but is standing on the equator, with the satellite in a low earth orbit moving directly east above the equator. He uses his own position as the reference point for measuring the period. Then when the satellite has passed through one complete ellipse, it will be behind the observer because the rotation of the earth will have carried him a distance eastward. The satellite will be over the observer again only after it has traveled an *additional* distance eastward. The period as now measured by the observer will obviously be greater than the sidereal period. It is called the *synodic* period.

The word *synodic* refers to a meeting or conjunction. At the beginning of the period, the position of

the spacecraft over the observer results in a certain grouping or meeting of the earth, spacecraft, and sun. At the end of the period the spacecraft will be over the observer again, and this same grouping or meeting will be repeated.

In practice, very few satellites are placed in equatorial orbits. Most orbits are inclined at an angle to the equator, as shown in Figure 2. In the case of an inclined orbit, the spacecraft will not make successive passes over the observer. The observer moves with the earth on a circle in a plane parallel to the plane of the equator, while the spacecraft moves through an ellipse in a plane inclined to the plane of the equator. Thus the point at which the spacecraft passes over the observer's longitude changes with each pass. For an inclined orbit, the time elapsing between two consecutive passes over the reference longitude is the synodic period.

In day-to-day operations, the practice arose of referring to the synodic periods simply as revolutions and the sidereal periods as orbits. The reader will find this terminology used in news accounts. Thus, astronauts Borman and Lovell completed 206 revolutions and 220 orbits during the 14-day mission of Gemini VII.

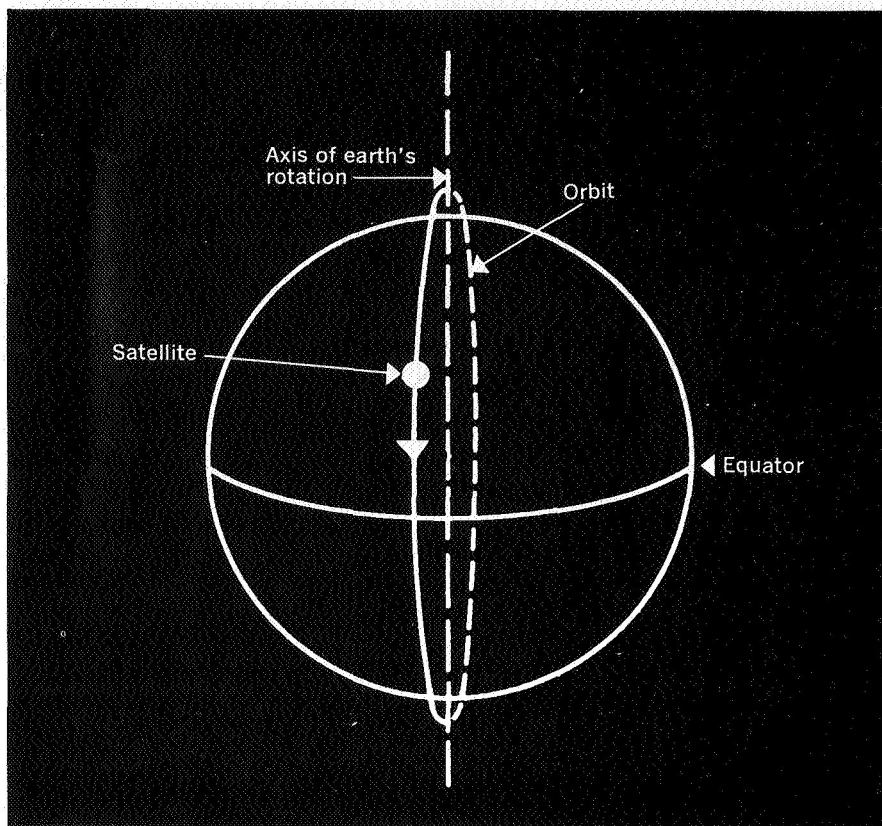


Figure 3

If the spacecraft is orbiting in an easterly direction, the same as the earth's rotation, the orbit is said to be *posigrade*, and the synodic period is greater than the sidereal period. If the spacecraft is revolving in a westerly direction, opposite to the earth's rotation, the orbit is said to be *retrograde*. In this case the eastward motion of the observer causes him to meet the satellite before it has completed a full transit of the ellipse, and the synodic period is therefore less than the sidereal period. All manned space flights launched to date by the United States have been placed in posigrade orbits in order to take advantage of the additional velocity imparted to the launch vehicle by the earth's rotation. Retrograde orbits require a greater expenditure of energy to attain orbital velocity, since the launch vehicle must overcome the velocity of rotation of the earth. If a spacecraft is placed in a polar orbit, as shown in Figure 3, the synodic and sidereal periods are equal.

The sidereal period can easily be computed through the use of the formula $P = 2\pi \sqrt{\frac{a^3}{GM}}$ in which a is the average radius of the orbit, G is the Constant of Universal Gravitation, and M is the mass of the earth, or other primary body. The value

of a is found by averaging the apogee and perigee distances as measured from the center of the earth. The apogee distance is the farthest distance of the spacecraft from the earth, while the perigee distance is the nearest. (Readers who have studied analytical geometry will recognize that a is the semi-major axis of the ellipse.) The reader can find a discussion of G and M , and of how they are measured, by referring to standard physics textbooks.

If our units of measurement are the statute mile for distance and the second for time, the numerical value of GM for orbits about the earth is $9.56 (10^4)$. We shall use this value, since most news reports of spacecraft orbits are given in English units. (If one uses metric units, he will have other values for a and GM , but the computed length of the period will be the same.)

Let us find the sidereal period of a satellite with an average altitude above the earth of 100 miles. This satellite is in an elliptical orbit for which the average of the apogee and perigee altitudes is 100 miles. Since the average radius of the earth is 3960 miles, $a = 3960 + 100 = 4060$. Then

$$P = 2(3.14) \sqrt{\frac{(4060)^3}{9.56 (10^4)}}$$

The solution is easily completed through the use of

a table of powers and roots. $P = 5256$ seconds approximately, which equals 87.6 minutes or 1.46 hours.

Computing the synodic period for an inclined orbit is beyond the scope of this discussion. However, for an equatorial orbit the synodic period can be found from the sidereal period by a rather simple computation. In Figure 4, let A be a position on the equator at which the satellite discussed above is directly over the observer. Then during one synodic period, the rotation of the earth carries the observer to B, where the satellite "overtakes" him again. Our basic problem is to find the angle θ .

If θ is the angular distance traveled by the observer during one synodic period, then $360 + \theta$ is the angular distance traveled by the satellite. The observer travels 360 degrees in 24 hours, or one degree in $\frac{24}{360}$ hours, and is "caught" by the satellite in $\frac{24}{360} \theta$ hours. Since the satellite travels one degree in $\frac{1.46}{360}$ hours, the time required for it to "catch" the observer is $\frac{1.46}{360} (360 + \theta)$. These two times are equal. Therefore,

$$\begin{aligned} \frac{1.46}{360} (360 + \theta) &= \frac{24}{360} \theta \\ 1.46 (360 + \theta) &= 24 \theta \\ 525.6 + 1.46 \theta &= 24 \theta \\ 22.54 \theta &= 525.6 \\ \theta &= 23.32 \text{ degrees} \end{aligned}$$

The synodic period is $\frac{1.46}{360} (360 + 23.32) = 1.555$ hours or 93.3 minutes. Thus the synodic period is $93.3 - 87.6 = 5.7$ minutes greater than the sidereal period.

ACTIVITIES

1. Find the synodic period of the above spacecraft if it is in a retrograde equatorial orbit. (82.6 min.)
2. A similar situation would be presented by two boys going around a circular track, one walking at

a speed of 200 feet per minute and the other riding a bicycle at a speed of 1000 feet per minute. Assume that the circumference of the track is 1000 feet. Solve the following problems when the boys are going in the same direction. In opposite directions.

- a. How long does it take each boy to complete one lap? (5 min., 1 min.)

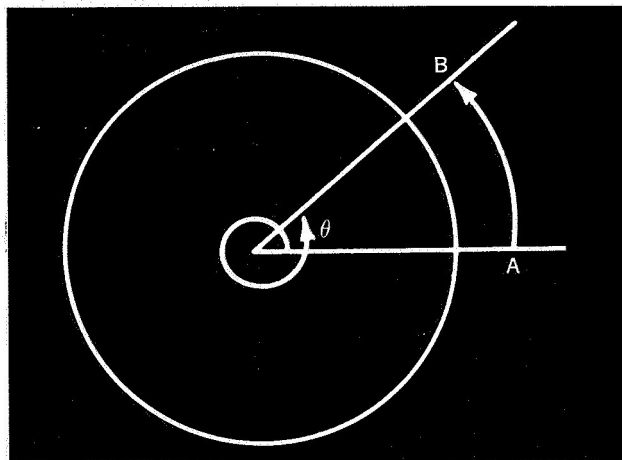


Figure 4

- b. How many laps does the riding boy make while the walking boy makes one? (5)
 - c. If the boys start together, how long will it take the riding boy to catch the walking boy? (75 sec., 50 sec.)
 - d. How far does the walking boy go between successive passes of the riding boy? (250 ft., 166.7 ft.)
3. Compute the sidereal periods of satellites at average altitudes above the surface of the earth of 500, 1000, and 22,300 miles. (101 min., 120 min., 24 hr.)
 4. The mass of the moon is .0123 times the mass of the earth, or .0123 M. Therefore the value of GM when applied to the moon is (9.56) (.0123) (10^4). Use this value to find the sidereal period of a lunar satellite whose average distance above the surface of the moon is 60 miles. The radius of the moon is 1080 miles. (118 min.)
 5. Readers who have studied physics may wish to compute the sidereal period of a satellite 100 statute miles above the earth by expressing the values of a, G, and M in the MKS system of units. The same length of period should be found.